

Evaporation from sparse crops—an energy combination theory

By W. JAMES SHUTTLEWORTH and J. S. WALLACE
Institute of Hydrology, Wallingford, Oxon.

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SUMMARY

A one-dimensional model is adopted to describe the energy partition of sparse crops. Theoretical development of this model yields a combination equation which describes evaporation in terms of controlling resistances associated with the plants, and with the soil or water in which they are growing. The equation provides a simple but physically plausible description of the transition between bare substrate and a closed canopy. Although the aerodynamic transfer resistances for incomplete canopies have, as yet, no experimental justification, typical values, appropriate to a specimen agricultural crop and soil, are shown to have limited sensitivity in the model. Processes which require further study if the equation is to be used to calculate evaporation throughout a crop season are also discussed.

1. INTRODUCTION

Previous steps in the development of a physically based model of the vegetation–atmosphere interaction (e.g. Shuttleworth 1976, 1978) explicitly treat the vegetation as a closed, stable canopy of uniform structure. They emphasize the interaction of the vegetation, with fluxes arising at the soil surface introduced as an unspecified, and implicitly small, input to the model (Shuttleworth 1979). In this paper this theoretical work is reinterpreted and developed into the situation of sparse crops, where the use of a one-dimensional model has less obvious justification. In describing such crops the soil and plant components must carry equal status, since they can be of similar size and their relative importance can change significantly with crop cover.

The philosophy of this paper is to make minimum concession to the more obvious three-dimensional structure of sparse and row crops. Accordingly a one-dimensional model of the interaction is adopted to derive a combination equation, which can provide a physically plausible transition between the bare substrate and closed canopy limits. The equation is expressed in terms of conceptual resistances now familiar to the micro-meteorologist and plant physiologist: canopy resistance and boundary layer resistance etc; it also requires the less familiar concept of a surface resistance for bare soil (Monteith 1981). In the later sections of the paper typical values of these resistances are used to illustrate how energy partition varies between crops of the same height, but with different leaf areas.

2. THE ONE-DIMENSIONAL MODEL

The problem of sparse crops is approached using the knowledge that the two asymptotic limits (bare substrate and a closed canopy) can both be represented by a one-dimensional model. Moreover it is acknowledged that, in describing the closed canopy situation, models which represent the canopy interaction as occurring as a single source (Monteith 1965) have increasing acceptance. Such models represent a practical compromise between physical rigour and field application. The basic assumption, that there is numerical similarity between bulk stomatal resistance and an integration of component stomatal resistances in dry conditions (Monteith 1965), has been tested experimentally (Black *et al.* 1970; Szeicz *et al.* 1973; Tan and Black 1976), numerically (Sinclair *et al.* 1971) and explored theoretically (Shuttleworth 1976).

Implicit in this successful assumption is the idea that the real three-dimensional

nature of a crop can be ignored in terms of its practical consequences. It is also assumed that aerodynamic mixing within the crop is sufficiently good to allow the hypothetical existence of a 'mean canopy airstream' (Thom 1972) which can be described by meteorological parameters such as temperature, humidity and wind speed. These assumptions are necessary if progress is to be made towards providing a description which is general (in the sense that it is not merely a description of a particular crop at a particular time). Previous models based on these assumptions have been proposed and tested (Black *et al.* 1970; Szeicz *et al.* 1973) even in closed canopies, where the aerodynamic interaction within the canopy is minimized.

The degree of aerodynamic mixing in sparse and row-structured crops is likely to be greater than that in closed canopies. Adopting the Monteith assumption for such crops is therefore arguably more plausible in terms of mixing in the vertical. It does, however, require a reconsideration of the scale relevant to the horizontal averaging process, and will involve additional uncertainty regarding the consequences of persistent features within the aerodynamic mixing pattern in the horizontal plane (see, for example, Arkin and Perrier 1974).

Any one-dimensional description assumes horizontal uniformity, but in practice recognizable three-dimensional features of the crop (individual plants and rows, and persistent aerodynamic mixing features) are always present. If a description of the Monteith type is to be used it is necessary that the elements of which the model is composed (e.g. energy fluxes, stomatal resistance, etc.) are defined as horizontal averages over area scales in which persistent features occur in sufficient numbers to allow such averaging. A one-dimensional description is clearly not relevant to horizontal scales less than this.

In this study a simple two-component structure is maintained and energy partition treated as occurring at 'the crop' and 'the soil'. This simplification appears particularly arbitrary in the case of row crops since it involves, for instance, no distinction between soil beneath the vegetation and that between the rows. The presence of a defined three-dimensional structure clearly cannot be totally ignored in certain aspects of the interaction. For example, in the case of interception of solar radiation, row orientation may affect the effective absorption coefficient relevant at the scale of the horizontal

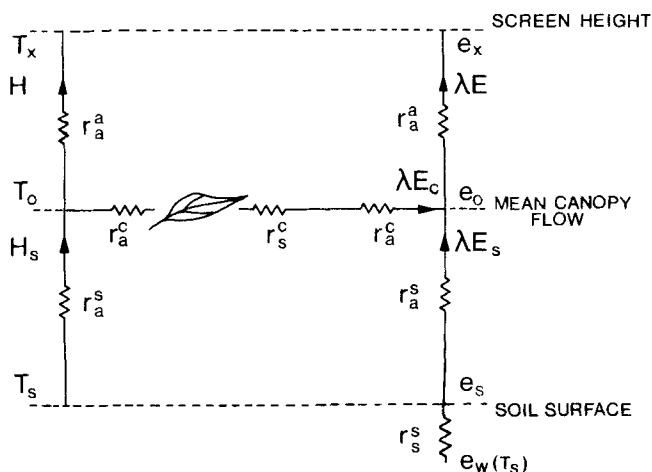


Figure 1. Schematic diagram of a one-dimensional description of energy partition for sparse crops. The nomenclature used is given in section 3(a).

averaging. The diurnal cycle in solar altitude reduces the distinction between the daily total absorption by such organized structures and that of randomly distributed sparse vegetation. Nonetheless, the realistic philosophy in dealing with row crops is to treat separately such aspects of the problem, but then to return to an averaging scale over which a one-dimensional model is assumed to apply.

The model adopted and developed in this paper is illustrated in Fig. 1. It incorporates not only the now familiar concept of a bulk stomatal resistance for the vegetation, r_s^c , but also the less familiar concept of a surface resistance at the substrate surface, r_s^s . The physical origin of this surface resistance is less obvious for soil than it is for vegetation but its *mathematical* definition is precise. Its presence reflects the fact that the layer of air adjacent to the soil surface is not necessarily saturated unless the soil surface is wet. If soil evaporation is λE_s , and the temperature and vapour pressure at the soil surface T_s and e_s respectively, then the surface resistance of the soil is defined by the equation

$$r_s^s = (\rho c_p / \gamma) \{e_w(T_s) - e_s\} / \lambda E_s \quad (1)$$

where $e_w(T_s)$ is the saturated vapour pressure at temperature T_s and the other quantities are defined in section 3(a). Monteith (1981) interprets this resistance in conceptual terms by describing evaporation from a drying soil as occurring from wet soil below a dry soil layer of increasing thickness, treated as isothermal. This provides a description qualitatively consistent with observation. Although this obvious over-simplification leads to problems in physical interpretation (Fuchs and Tanner 1967), Eq. (1) defines an entity which can form the subject of empirical models. Such modelling is not discussed further in the present paper.

The model described in Fig. 1 adopts the concept of a bulk boundary layer resistance, r_a^c , which controls transfer between the surface of the vegetation and the canopy air stream (Thom 1972). Vertical transport is controlled by two further aerodynamic resistances. The first, r_a^a , is the transfer resistance between the hypothetical mean canopy flow and the reference height, x , above the crop. The second, r_a^s , is the aerodynamic resistance encountered by the energy fluxes leaving the substrate before they are incorporated into the mean canopy flow. For simplicity in this analysis it is assumed that the various aerodynamic resistances are identical for sensible and latent heat. Making this assumption simplifies the formalism but does not alter the derivation in any fundamental way.

It is worth remembering at this point that the so called 'aerodynamic resistance' generally employed when describing energy partition with a combination equation (Monteith 1965) is obtained by the addition of the component resistances described in the previous paragraph. In this way the aerodynamic resistance used in a combination equation describing a closed canopy (with no soil evaporation) is $r_a^a + r_a^c$, while that which would be used to describe evaporation from the substrate is $r_a^a + r_a^s$. The relevant 'surface' resistances in these two situations are r_s^c and r_s^s . The objective in the following sections is to derive a combination equation, descriptive of both plant and substrate evaporation, but which asymptotes towards simpler combination equations involving the aerodynamic and surface resistances relevant in limiting situations.

3. THEORETICAL DEVELOPMENT

(a) Nomenclature

- A, A_s Total energy flux leaving the complete crop, the substrate, as sensible and latent heat per unit ground area (W m^{-2})
- C Extinction coefficient of the crop for net radiation (dimensionless)

c_p	Specific heat at constant pressure ($\text{J kg}^{-1}\text{K}^{-1}$)
d	Zero plane displacement of crop with complete canopy cover ($L = 4$) (m)
D	Vapour pressure deficit at reference height, $e_w(T_x) - e_x$ (mb)
D_0	Vapour pressure deficit at canopy source height, $e_w(T_0) - e_0$ (mb)
e_0, e_s	Vapour pressure at canopy source height, soil surface, (mb)
e_x	Vapour pressure at reference height (mb)
$e_w(T)$	Saturated vapour pressure at temperature T ($T = T_x, T_0, T_s$) (mb)
G	Soil heat flux (W m^{-2})
h	Crop height (m)
H, H_s	Sensible heat flux from the complete crop, substrate, (W m^{-2})
k	von Kármán's constant (dimensionless)
K	Eddy diffusion coefficient (m^2s^{-1})
K_h	Eddy diffusion coefficient at top of canopy (m^2s^{-1})
L	Projected area of leaf per unit ground area (leaf area index) (dimensionless)
n	Eddy diffusivity decay constant in a crop with complete canopy cover ($L = 4$) (dimensionless)
P	Biochemical storage of energy in the crop below reference height (W m^{-2})
r_a^a	Aerodynamic resistance between canopy source height and reference level (s m^{-1})
r_a^c	Bulk boundary layer resistance of the vegetative elements in the canopy (s m^{-1})
r_a^s	Aerodynamic resistance between the substrate and canopy source height (s m^{-1})
r_b	Mean boundary layer resistance per unit area of vegetation (s m^{-1})
r_s^c	Bulk stomatal resistance of the canopy (s m^{-1})
r_s^s	Surface resistance of the substrate (s m^{-1})
r_{ST}	Mean stomatal resistance (s m^{-1})
$r_a^a(0)$	Value of r_a^a for bare substrate (s m^{-1})
$r_a^a(\alpha)$	Value of r_a^a for crop with complete canopy cover ($L = 4$) (s m^{-1})
$r_a^s(0)$	Value of r_a^s for bare substrate (s m^{-1})
$r_a^s(\alpha)$	Value of r_a^s for crop with complete canopy cover ($L = 4$) (s m^{-1})
R_n	Net radiation flux into the complete crop (W m^{-2})
R_n^s	Net radiation flux into the substrate (W m^{-2})
S	Physical storage of energy in the atmosphere and crop below reference height (W m^{-2})
T_0	Air temperature at canopy source height ($^{\circ}\text{C}$)
T_s	Temperature of the substrate surface ($^{\circ}\text{C}$)
T_x	Air temperature at reference height ($^{\circ}\text{C}$)
u	Wind speed at the reference height (m s^{-1})
u_*	Friction velocity (m s^{-1})
x	Reference height above the crop where meteorological measurements are available (2 m in this analysis)
z	Height (variable) (m)
z_0	Roughness length of crop with complete canopy cover ($L = 4$) (m)
z_0'	Roughness length of the bare substrate (m)
Δ	Mean rate of change of saturated vapour pressure with temperature, $\{e_w(T_x) - e_w(T_0)\}/(T_x - T_0)$ (mb K^{-1})
γ	Psychrometric 'constant' (mb K^{-1})
λE	Latent heat flux from the complete crop (W m^{-2})
λE_c	Latent heat flux from the plant canopy (W m^{-2})

λE_s Latent heat flux from the substrate (W m^{-2})
 ρ Density of air (kg m^{-3})

(b) *The energy budget*

All combination equations rely on drawing up an energy budget between the outward fluxes of sensible and latent heat and the energy available in other forms. In this problem two budgets are drawn up, one at the substrate surface and one for the complete crop. The sum of the above-canopy fluxes of sensible heat, H , and latent heat, λE , is the available energy, A , and is given by

$$A = \lambda E + H \quad (2)$$

$$= R_n - S - P - G \quad (3)$$

where R_n is the incoming net radiation, S and P are the physical and biochemical energy storage terms, and G is the heat conduction into the substrate. In a similar way, the energy available at the substrate, A_s , is given by

$$A_s = \lambda E_s + H_s \quad (4)$$

$$= R_n^s - G \quad (5)$$

where R_n^s is the net radiation at the substrate surface.

In drawing up such energy budgets it is clearly necessary to consider average values of the several components defined over horizontal scales which involve significant numbers of the identifiable crop and soil features. In general R_n^s is less than R_n and A_s less than A . In the limit of bare substrate A and A_s are equal.

(c) *In-canopy deficit*

By analogy with Ohm's law for the electrical analogue shown in Fig. 1, the difference in vapour pressure and temperature between the level of mean canopy flow and reference height can be written in terms of resistance and flux as

$$e_x - e_0 = -\lambda E r_a^a \gamma / \rho c_p \quad (6)$$

and

$$T_x - T_0 = -H r_a^a / \rho c_p \quad (7)$$

Introducing the definition of Δ into the expression for the vapour pressure deficit at the canopy source height, D_0 , gives

$$D_0 = e_w(T_x) - \{e_w(T_x) - e_w(T_0)\} - e_0$$

and substituting Eqs. (7), (6) and (2), yields a relationship between D_0 and D such that

$$D_0 = D + \{\Delta A - (\Delta + \gamma)\lambda E\} r_a^a / \rho c_p \quad (8)$$

(d) *The sparse crop combination equation*

In the model illustrated (Fig. 1), the evaporation from the substrate, λE_s , and from plants in the canopy, λE_c , can be separately calculated from equations of the Penman-Monteith type, thus

$$\lambda E_s = (\Delta A_s + \rho c_p D_0 / r_a^s) \{\Delta + \gamma(1 + r_s^s / r_a^s)\}^{-1} \quad (9)$$

$$\lambda E_c = \{\Delta(A - A_s) + \rho c_p D_0 / r_a^c\} \{\Delta + \gamma(1 + r_s^c / r_a^c)\}^{-1} \quad (10)$$

The total evaporation from the crop, λE , is the sum of these two, and it can be shown (see appendix) that D_0 can be eliminated and the resultant equation arranged in the form

$$\lambda E = C_c PM_c + C_s PM_s \quad (11)$$

where PM_c and PM_s are terms each similar to the Penman–Monteith combination equations which would apply to evaporation from a closed canopy and from bare substrate respectively. They have the form

$$PM_c = \frac{\Delta A + \{\rho c_p D - \Delta r_a^c A_s\} / (r_a^a + r_a^c)}{\Delta + \gamma \{1 + r_s^c / (r_a^a + r_a^c)\}} \quad (12)$$

$$PM_s = \frac{\Delta A + \{\rho c_p D - \Delta r_a^s (A - A_s)\} / (r_a^a + r_a^s)}{\Delta + \gamma \{1 + r_s^s / (r_a^a + r_a^s)\}} \quad (13)$$

The coefficients C_c and C_s are given by the expressions

$$C_c = \{1 + R_c R_a / R_s (R_c + R_a)\}^{-1} \quad (14)$$

and

$$C_s = \{1 + R_s R_a / R_c (R_s + R_a)\}^{-1} \quad (15)$$

where

$$R_a = (\Delta + \gamma) r_a^a \quad (16)$$

$$R_s = (\Delta + \gamma) r_a^s + \gamma r_s^s \quad (17)$$

$$R_c = (\Delta + \gamma) r_a^c + \gamma r_s^c \quad (18)$$

It can be easily seen that Eq. (11) has correctly defined asymptotic limits. If there is no substrate evaporation, r_s^s and hence R_s are infinite, PM_s is zero in Eq. (11), and C_c is unity. If, in addition, there is no sensible heat flux from the substrate, H_s and hence A_s are zero, and Eq. (11) reduces to the conventional Penman–Monteith equation describing closed canopy evaporation with no substrate interaction. In a similar way, if there is no canopy present r_s^c and hence R_c are infinite, and $A = A_s$. Equation (11) reduces to a conventional form, describing substrate evaporation with the Penman–Monteith equation involving a surface resistance applicable to the substrate.

In the more general intermediate situation, when both substrate and canopy evaporation occur, Eq. (11) provides a physically plausible description of the total evaporation λE . Once calculated, this can be substituted into Eq. (8) to compute D_0 , and in this way the component fluxes λE_c and λE_s calculated from Eqs. (9) and (10) if required.

(e) *The leaf area dependence of surface resistance*

The assumptions and observations made in Shuttleworth (1979) and associated papers (Shuttleworth 1976; 1978) are implicit in Eq. (11). In the context of the present problem a point of particular relevance is the fact that the mean boundary layer resistance of the canopy, r_a^c , and the bulk stomatal resistance of the canopy, r_s^c , are both 'surface' resistances, influenced by the surface area of the vegetation present. They vary inversely with the total leaf area of the vegetative elements present (Shuttleworth 1976). This is important when considering the effect of changes in leaf area index on energy partition and it is convenient in this case to rewrite these two surface resistances in the form

$$r_s^c = r_{ST} / 2L \quad (19)$$

$$r_a^c = r_b / 2L \quad (20)$$

where r_{ST} is the mean stomatal resistance (of amphistomatous leaves) and r_b the mean boundary layer resistance, both expressed per unit surface area of vegetation. L is the leaf area index of the canopy, i.e. the projected area of the vegetation per unit ground area. In this way these two resistances are high in very sparse canopies and, all other things being equal, decrease inversely as the area of vegetation per unit ground area increases. In practice this is just a first approximation since r_b may itself be influenced by changes in mean canopy wind speed, and r_{ST} may change in response to differences in shading.

The resistance r_s^s is also a surface resistance and should therefore be divided by the area of exposed substrate per unit ground area. In practice the area occupied by plant stems is likely to be a small fraction of ground area. Any litter present on the ground can be regarded as part of the substrate.

4. VEGETATION DENSITY VARIATIONS

In this section the response of Eq. (11) is explored when applied to crops of the same height but with different leaf areas, which are subject to a specified atmospheric demand. For the purposes of comparison, calculations are carried out assuming *measured* meteorological variables are available above the crops at a height, x , of 2 m. In this way it is assumed that any density-related interaction between the crop and the meteorological variables, e.g. albedo changes, can be treated separately (we return to this point later). Calculations are made for soil and water substrates, using a specification of the component resistances drawn from the literature, and taken as applying to a 'typical' agricultural crop.

(a) Model specification

(i) *Available energy.* The physical and biochemical storage terms, S and P , are ignored in Eq. (3). Since net radiation during daylight hours is primarily determined by direct radiation, it has been found experimentally (e.g. Ross 1981) that the radiation reaching the soil surface, R_n^s , can be calculated using a Beer's law relationship of the form

$$R_n^s = R_n \exp(-CL) \quad (21)$$

where C is the extinction coefficient of the crop for net radiation, chosen arbitrarily as 0.7 (see for example Monteith 1973). It is convenient here to ignore variations in C which may occur in response to structural differences in crops of different density, although this could (if known) be included in specific cases. In the present calculation the heat conduction into the substrate, G , is arbitrarily set to 20% of the radiation received at the substrate surface, R_n^s , and therefore G also changes with crop density.

(ii) *Mean stomatal resistance.* The mean stomatal resistance, r_{ST} , is taken as 400 s m^{-1} . It follows from Eq. (19) that, for a leaf area index, L , of 4, the bulk stomatal resistance, r_s^s , is 50 s m^{-1} . This value is typical of a fully grown agricultural crop (see Wallace *et al.* 1981).

(iii) *Mean boundary layer resistance.* Measurements of mean boundary layer resistance, r_b , generally have significant scatter and exhibit some dependence on in-canopy wind speed. The value of r_b for stands of vegetation of different density is therefore uncertain. Typical values measured in the field are in the order 25 s m^{-1} (see Denmead 1976; Uchijima 1976) and this value is assumed here. The corresponding bulk boundary layer resistance is 3 s m^{-1} for a leaf area index of $L = 4$. In practice this resistance is only significant in Eq. (11) when acting in combination with the much larger bulk stomatal

resistance; errors in its assumed value are shown later to be of limited numerical importance.

(iv) *Surface resistance of the substrate.* The description of substrate evaporation in terms of a surface resistance is somewhat novel in this paper, and, in consequence, typical values are difficult to specify. Calculations are performed for three values of r_s^s : 0, 500 and 2000 s m^{-1} . The first value, 0, corresponds to a substrate of wet soil or free water, whilst the last, 2000, is arguably typical of fairly dry soil—theoretically, it corresponds to molecular diffusion through a 1.5 cm thick layer of dry sandy soil (Fuchs and Tanner 1967). The third value, 500, is chosen merely as an intermediate value; on a unit area basis it is in the same order as that of dry vegetation.

(v) *Eddy diffusion resistance.* Clearly crop density affects the size of the aerodynamic resistances r_a^a and r_a^s , but the quantitative response of within-canopy aerodynamic transfer to differing leaf area index is perhaps the least understood aspect of micrometeorology, and likely to remain so in the foreseeable future. In these circumstances we have assumed the simplest possible model in which r_a^a and r_a^s vary linearly with leaf area index between the values associated with their two limits, namely bare substrate and a complete canopy cover (which we arbitrarily assume corresponds to $L = 4$). In fact, as we demonstrate later, the quantitative effect of this simple treatment has a limited effect on component and total evaporation fluxes.

In crops with complete cover the effective source, at which mean canopy air stream conditions are assumed to apply, is defined to occur at a height $(d + z_0)$ in the crop, where d is zero plane displacement and z_0 is crop roughness length. Monteith (1973) relates d and z_0 to crop height for the fully developed crop through the expressions

$$d = 0.63h \quad z_0 = 0.13h. \quad (22)$$

In sparser crops we assume that the effective source height remains fixed at this fraction of crop height. It follows that in this exercise, where crop height is held constant, the effective height at which the plant components of sensible and latent heat arise is assumed to be independent of crop density. For simplicity, stability effects are ignored here and the eddy diffusion coefficients describing the vertical movement of heat and water vapour are equated to those for momentum.

Above the fully developed crop ($L \geq 4$), the eddy diffusion coefficient, K , is given by

$$K = ku_*(z - d) \quad (z > h) \quad (23)$$

where k is von Kármán's constant, z is height and u_* is the friction velocity, which, in conditions of neutral atmospheric stability, is given by the expression

$$u_* = ku/\ln\{(x - d)/z_0\} \quad (24)$$

where u is the wind speed at the reference height x . It is also assumed that, in the closed canopy, the eddy diffusion coefficient decreases exponentially with height, thus

$$K = K_h \exp\{-n(1 - z/h)\} \quad (25)$$

where K_h is the value of K at the top of the crop, $ku_*(h - d)$; we use a value of $n = 2.5$ which is typical of the agricultural crop being specified (Monteith 1973). Using these several assumptions and performing an integration over the height ranges 0 to $d + z_0$ and $d + z_0$ to x respectively, it is possible to write

$$r_a^s(\alpha) = \frac{\ln\{(x - d)/z_0\}}{k^2 u} \frac{h}{n(h - d)} [\exp n - \exp\{n[1 - (d + z_0)/h]\}] \quad (26)$$

$$r_a^a(\alpha) = \frac{\ln\{(x-d)/z_0\}}{k^2u} \left[\ln\{(x-d)/(h-d)\} + \frac{h}{n(h-d)} \times \right. \\ \left. \times [\exp\{n\{1 - (d+z_0)/h\}\} - 1] \right]. \quad (27)$$

For a crop height (h) of 0.3 m and wind speed (u) of 2 m s^{-1} , this gives $r_a^s(\alpha) = 128 \text{ s m}^{-1}$ and $r_a^a(\alpha) = 42 \text{ s m}^{-1}$.

With a bare substrate, computation is simpler. Here the aerodynamic resistances are given by

$$r_a^s(0) = \ln(x/z'_0) \ln\{(d+z_0)/z'_0\} / k^2u \quad (28)$$

$$r_a^a(0) = \ln^2(x/z'_0) / k^2u - r_a^s(0) \quad (29)$$

where z'_0 is the effective roughness length of the substrate. For bare soil z'_0 is commonly taken as 0.01 m (see Van Bavel and Hillel 1976) and for simplicity here differences in the surface roughness between wet soil and free water substrates are neglected. The values of these resistances in the present situation with $u = 2 \text{ m s}^{-1}$ are $r_a^s(0) = 49 \text{ s m}^{-1}$ and $r_a^a(0) = 34 \text{ s m}^{-1}$ respectively.

Since we do not yet know exactly how r_a^a and r_a^s will vary, we assume they have a linear relationship between their asymptotic limits. Thus

$$\left. \begin{aligned} r_a^a &= \frac{1}{4}Lr_a^a(\alpha) + \frac{1}{4}(4-L)r_a^a(0) \\ r_a^s &= \frac{1}{4}Lr_a^s(\alpha) + \frac{1}{4}(4-L)r_a^s(0) \end{aligned} \right\} \quad 0 \leq L \leq 4 \quad (30)$$

$$\left. \begin{aligned} r_a^a &= r_a^a(\alpha) \\ r_a^s &= r_a^s(\alpha) \end{aligned} \right\} \quad L > 4 \quad (31)$$

It is shown later that the exact forms of these relationships are usually of limited numerical importance in calculations of evaporation. Implicit in the above equations is the assumption that the roughness length and zero plane displacement of crops with intermediate cover ($0 < L < 4$) vary between the values appropriate to complete cover (z_0 and d) and bare soil (z'_0 and zero) and are therefore *not* a fixed fraction of crop height. However, the effective source height of the energy fluxes from the vegetation (mean canopy flow) is assumed to be a fixed fraction of crop height.

(b) Model predictions

Calculations are carried out for the following meteorological conditions: $R_n = 400 \text{ W m}^{-2}$; $D = 0, 10, 20 \text{ mb}$; $T_x = 25^\circ\text{C}$; and $u = 2 \text{ m s}^{-1}$. Such meteorological conditions might be considered typical for midday in the middle of a growing season at a subtropical site. However, the objective is not to make detailed predictions for particular meteorological conditions, it is rather to illustrate the general features of the theoretical treatment described.

(i) *Free water substrate.* The situation in which $r_s^s = 0$ is in some regards a particular case. It might be considered to represent the behaviour of paddy rice or crops which are (over) watered by trickle irrigation. Results for this situation are illustrated in Fig. 2.

Figure 2(a) illustrates total crop evaporation rates for the meteorological conditions specified above, for crops of different density, defined by their leaf area index. These

rates are compared with the energy available to the whole crop, and that available to the substrate. The energy available to the system increases slightly with crop cover, while that of the substrate falls monotonically. This behaviour merely reflects the assumptions regarding G and R_n^s made in section 4(a)(i). The total evaporation rate varies considerably with vapour pressure deficit, but for a given atmospheric demand is fairly independent of crop cover ($\pm 9\%$). The rate dips around $L = 1$ when radiation capture by the plants is significant, but their bulk stomatal resistance is still quite large.

Figure 2(b) illustrates the fraction of the available energy partitioned by the vegetation, $(A - A_s)/A$, and the fractional contribution made to total evaporation by the plants, $\lambda E_c/\lambda E$, for different atmospheric deficits. The fraction of the total evaporation generated by the crop is fairly insensitive to deficit and noticeably less than the fraction of radiation intercepted.

(ii) *Substrate resistance dependence.* Figure 3 illustrates the effect of changes in the surface resistance of a soil substrate for crops of different density. Calculations are presented for an atmospheric vapour pressure deficit of 20 mb; the effect at different deficits is qualitatively similar, although the actual value of the evaporation rate changes.

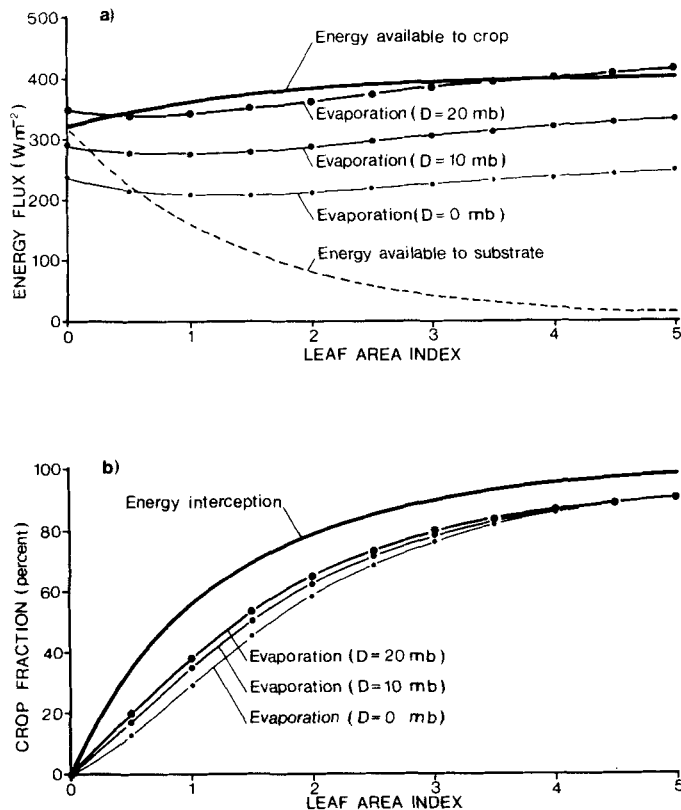


Figure 2. (a) Energy available to the crops and their free water substrates expressed as a function of L , compared with computed total crop evaporation rates for the model and conditions described in the text, with vapour pressure deficits of 0, 10 and 20 mb

(b) Fraction of total evaporation originating from the plants expressed as a function of L , computed for the model and conditions described in the text, with vapour pressure deficits of 0, 10 and 20 mb and a free water substrate. The fraction of energy intercepted by the vegetation is also shown for comparison. (R_n , T_x , u , C , n , r_s^s , r_{ST} , r_b , x , h and z_0^s held constant)

The total evaporation rate of sparse crops is significantly altered by the condition (i.e. surface resistance) of the soil substrate (Fig. 3(a)). The contribution to total evaporation made by plants is also sensitive to r_s^s , and can easily exceed the fraction of energy intercepted by the canopy when leaf area index is low ($L < 2$) and soil surface resistance high. In this situation some of the energy incident on the soil is transferred as sensible heat to the canopy and utilized there for transpiration.

(c) Model sensitivity

In this section the sensitivity of the calculations made using the sparse crop combination equation (Eq. (11)) to the assumptions made in section 4(a) is explored when the model is used with surface resistances appropriate to sparse crops growing in soil.

(i) *The parametrization of aerodynamic resistance.* Calculations of evaporation rate and the fraction of evaporation arising in the crop are presented in Tables 1 and 2 respectively. The values presented are for a vapour pressure deficit of 20 mb, a stomatal resistance of 400 s m^{-1} and a soil surface resistance of 500 s m^{-1} with the assumptions made in 4(a), except that extreme changes are made in the parametrization of aerodynamic resistance.

Tables 1(a) and 2(a) illustrate the effect of halving and doubling the assumed value of mean boundary layer resistance. Clearly the model, and the physical process it

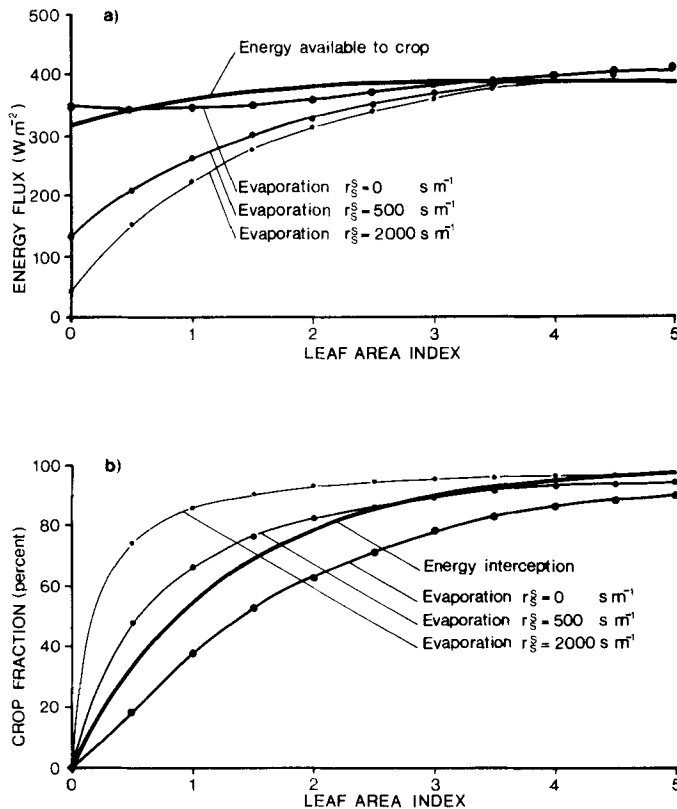


Figure 3. (a) Computed total crop evaporation rates expressed as a function of L for the model and conditions described in the text, with substrate surface resistances of 0, 500 and 2000 s m^{-1} , compared with the energy available to the crop.

(b) Fraction of total evaporation originating from the plants expressed as a function of L computed for the model and conditions described in the text with substrate surface resistances of 0, 500 and 2000 s m^{-1} . (R_n , T_x , u , D , C , n , r_{sT} , r_b , x , h and z'_0 held constant)

TABLE 1. CALCULATED TOTAL CROP EVAPORATION RATES ($W m^{-2}$) FOR $D = 20 mb$, $r_s^s = 500 s m^{-1}$ AND $r_{ST} = 400 s m^{-1}$ WITH CHANGES IN THE PARAMETRIZATION OF AERODYNAMIC RESISTANCE (SEE TEXT).

Model change	Leaf area index						
	0	0.5	1.0	1.5	2.0	3.0	4.0
(a) $r_b = 12.5 s m^{-1}$	135	210	263	302	331	370	394
No change	135	209	261	300	329	368	392
$r_b = 50 s m^{-1}$	135	207	259	297	325	364	387
	(0)	(1.4)	(1.5)	(1.7)	(1.8)	(1.6)	(1.8)
(b) $n = 1.25$	135	207	259	299	329	370	396
No change	135	209	261	300	329	368	392
$n = 5.0$	135	223	270	303	328	362	382
	(0)	(7.7)	(4.2)	(1.3)	(0.3)	(2.2)	(3.6)
(c) Cover	164	221	265	299	326	365	392
No change	135	209	261	300	329	368	392
Bare	135	206	259	300	332	378	409
	(21.5)	(7.2)	(2.3)	(0.3)	(1.8)	(3.5)	(4.3)

Numbers in brackets are the full range difference in the two perturbed rates expressed as a percentage of the rate given by the unmodified model.

TABLE 2. FRACTION OF TOTAL EVAPORATION ORIGINATING FROM THE PLANTS (PER CENT) CALCULATED FOR $D = 20 mb$, $r_s^s = 500 s m^{-1}$ AND $r_{ST} = 400 s m^{-1}$ WITH CHANGES IN THE PARAMETRIZATION OF AERODYNAMIC RESISTANCE (SEE TEXT).

Model change	Leaf area index						
	0	0.5	1.0	1.5	2.0	3.0	4.0
(a) $r_b = 12.5 s m^{-1}$	0	48.2	66.7	76.6	82.8	89.7	93.3
No change	0	47.9	66.5	76.4	82.5	89.5	93.2
$r_b = 50 s m^{-1}$	0	47.4	66.0	75.9	82.1	89.2	92.9
	(0)	(1.7)	(1.0)	(0.9)	(0.8)	(0.6)	(0.4)
(b) $n = 1.25$	0	48.7	67.3	76.9	82.6	88.8	92.0
No change	0	47.9	66.5	76.4	82.5	89.5	93.2
$n = 5.0$	0	43.3	62.5	74.4	82.3	91.2	95.4
	(0)	(11.3)	(7.2)	(3.3)	(0.4)	(2.7)	(3.6)
(c) Cover	0	44.4	64.6	75.7	82.4	89.7	93.2
No change	0	47.9	66.5	76.4	82.5	89.5	93.2
Bare	0	48.7	67.3	76.9	82.6	88.8	91.9
	(0)	(9.0)	(4.1)	(1.6)	(0.2)	(1.0)	(1.4)

Numbers in brackets are the full range difference in the two perturbed values expressed as a percentage of the fraction given by the unmodified model.

describes, are rather insensitive to the value of r_b : changing the value by a factor two changes λE and $\lambda E_c/\lambda E$ by 2% or less.

The effect of halving and doubling the constant n , which describes the exponential decay in eddy diffusivity through a fully developed crop, Eq. (25), is illustrated in Tables 1(b) and 2(b). The magnitude of the response is in the order of 5%, the proportionately largest effect in sparse canopies.

Tables 1(c) and 2(c) test the sensitivity of the model to extreme changes in the parametrization of r_a^a and r_a^s as a function of the leaf area index, L , given in Eqs. (30) and (31). Calculations are made with these aerodynamic resistances held at their complete cover and bare substrate limits. Again the magnitude of the response is typically in the

order of 5%. The effect on total evaporation rate of using the complete cover resistance in bare substrate conditions is greater than this, 21.5%, but corresponds to miscalculating the aerodynamic resistance by a factor of two in conditions of high vapour pressure deficit.

(ii) *Net radiation absorption.* Table 3 presents calculations of (a) evaporation rate and (b) the fraction of evaporation originating from the crop for $D = 20$ mb, $r_{ST} = 400 \text{ s m}^{-1}$ and $r_s^s = 500 \text{ s m}^{-1}$, with other parameters as in section 4(a) except that C is altered. Calculations are made for $C = 0.7$, as elsewhere in the analysis, and for $C = 0.5$ and 0.9 . Such a range in radiation absorption coefficient is not atypical of that found for real crops. With these assumptions the response of total evaporation rate to changes in C is small, less than 1%; that in the plant fraction of this evaporation is larger, 5–10%.

Some care is necessary in interpreting these particular results. The calculations presented in Table 3 represent the behaviour of sparse crops growing in soil with a surface resistance $r_s^s = 500 \text{ s m}^{-1}$ and a stomatal resistance $r_{ST} = 400 \text{ s m}^{-1}$ (consistent with the rest of this section). However, in this situation we get a minimal response of evaporation to changes in C . This parameter directly controls the fractional absorption of radiation by the plants and is therefore a driving mechanism in the initial routing of energy for partition by the surface resistances of the plants and soil. In this calculation r_s^s and r_{ST} are in the same order and this tends to suppress the response of the total evaporation rate to changes in C . Changes in the fraction of evaporation originating from the plants are more affected by changes in C .

(iii) *Mean stomatal resistance.* Figure 4 illustrates how the value of r_{ST} , and hence r_s^c , controls transpiration in crops with different density. The total evaporation rate changes significantly, though not of course proportionally, when r_{ST} is halved and doubled, and the proportion of evaporation originating at the plants also changes, especially in the sparser crops.

5. CONCLUDING REMARKS

The assumption (made in section 4(a)(v)) that the effective source height of the crop component of energy flux remains fixed at the value of $z_0 + d$ relevant to complete cover,

TABLE 3. (a) TOTAL CROP EVAPORATION RATE (W m^{-2}) AND (b) FRACTION OF TOTAL EVAPORATION ORIGINATING FROM THE PLANTS (PERCENT), CALCULATED FOR $D = 20$ mb, $r_s^s = 500 \text{ s m}^{-1}$ AND $r_{ST} = 400 \text{ s m}^{-1}$, FOR CHANGES IN THE NET RADIATION EXTINCTION COEFFICIENT $C = 0.5, 0.7, 0.9$.

Calculated parameter	Leaf area index						
	0	0.5	1.0	1.5	2.0	3.0	4.0
(a) Evaporation							
$C = 0.5$	135	208	260	298	327	366	390
$C = 0.7$	135	209	261	300	329	368	392
$C = 0.9$	135	209	262	301	330	369	392
	(0)	(0.5)	(0.8)	(1.0)	(0.9)	(0.8)	(0.5)
(b) Plant fraction							
$C = 0.5$	0.0	45.5	63.2	72.9	79.1	86.7	91.0
$C = 0.7$	0.0	47.9	66.5	76.4	82.5	89.5	93.2
$C = 0.9$	0.0	50.1	69.1	78.9	84.8	91.1	94.1
	(0)	(9.6)	(8.9)	(7.9)	(6.9)	(4.9)	(3.3)

Numbers in brackets are the full range difference in the two perturbed values expressed as a percentage of those given with $C = 0.7$.

irrespective of crop density, should not be misinterpreted as implying that z_0 and d are themselves assumed constant with changing crop density. These roughness parameters do of course change in the present model and tend to the values for bare soil (z'_0 and zero) in the limit. The assumption made here is, rather, that the plant components of the energy fluxes can be considered as arising at a particular fraction of the crop height

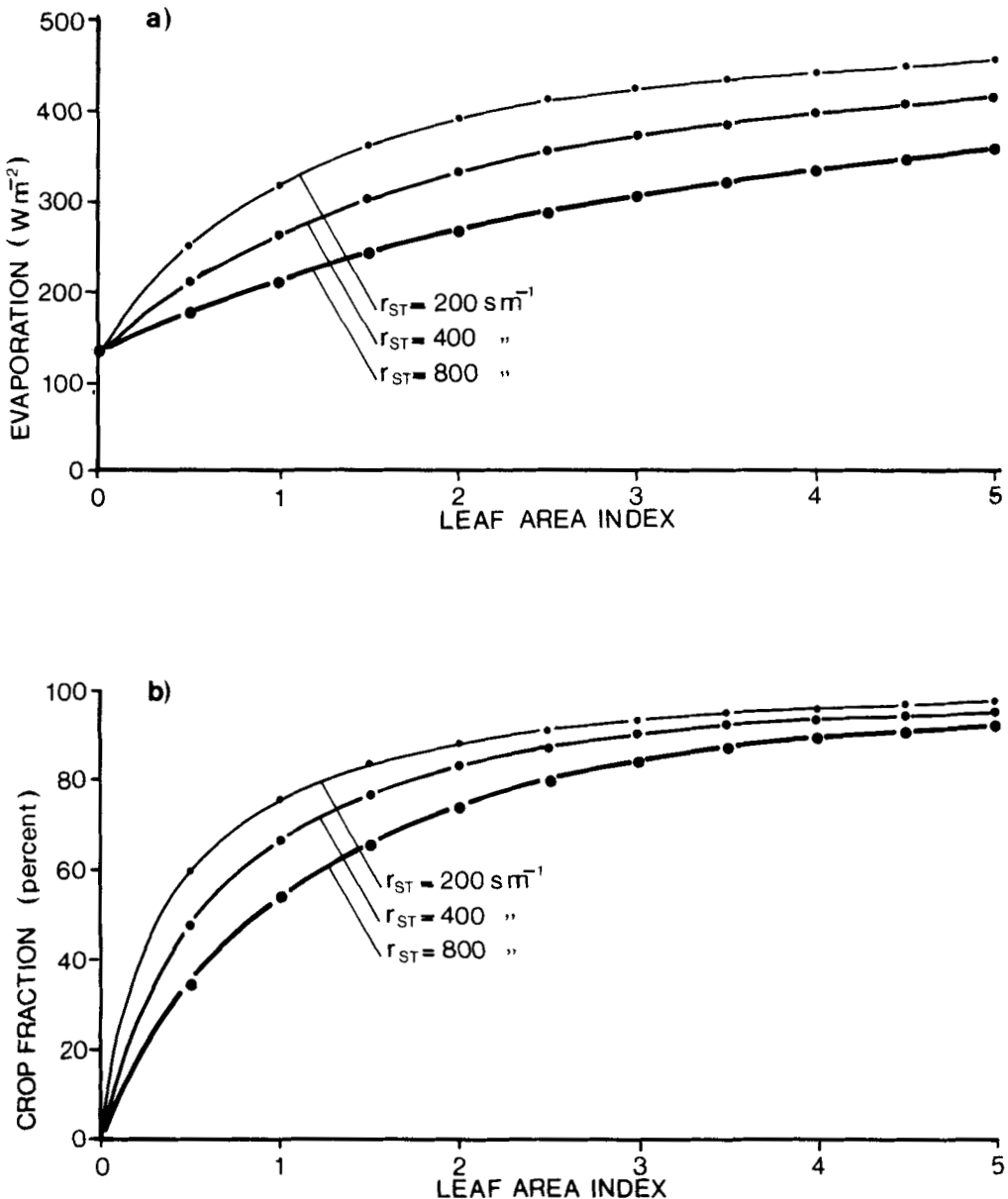


Figure 4. (a) Computed total crop evaporation rates expressed as a function of L for the model and conditions described in the text with mean stomatal resistances of 200, 400 and $800 s m^{-1}$. (b) Fraction of total evaporation originating from the plants expressed as a function of L computed for the model and conditions described in the text with mean stomatal resistances of 200, 400 and $800 s m^{-1}$. ($R_n, T_x, u, D, C, n, r_s^s, r_b, x, h$ and z'_0 held constant)

($0.76h$; Eq. (22)), which is specified by the value of $z_0 + d$ relevant to a closed canopy, and remains fixed at this fraction of h as vegetation density changes.

The assumption that the crop flux source height is a constant fraction of crop height involves approximation, as indeed does the additional implicit assumption that a single source level is appropriate for both heat and vapour for all crop densities, and that this is coincident with the effective sink of momentum in a closed canopy. Moreover, we have chosen to simplify the present calculations and presentation by ignoring the effect of stability in calculating components of aerodynamic resistance (although it would be fairly simple to include an iterative correction to allow for these using empirical stability functions above the crop or soil). These several approximations in the present model serve to exacerbate and illustrate a more basic lack of understanding of how aerodynamic transfer resistances evolve as crops grow. In the light of this, the limited sensitivity of the present theoretical description to extreme changes in the parametrization of aerodynamic resistance (see section 4(c)(i)) is an important feature of the present paper.

The current example calculations of the variation in energy partition with crop density at fixed crop height (presented in sections 4(b) and (c)) should not be regarded as describing the variation in energy partition as a crop grows, even though this may eventually be the most likely use of the model presented here. The aim of the present paper is to suggest a mathematical scheme which can be used to calculate evaporation when measurements or submodels of the crop height, leaf area, stomatal and substrate resistance, net radiation interception and soil heat flux are available. The difficulties involved in providing such crop-specific submodels are not underestimated. Some are clearly interrelated, for example, leaf area influences soil heat flux and net radiation (through albedo); and, apart from the direct effect on bulk stomatal resistance, leaf area could also have indirect effects on biological control by changing the radiation loading on individual leaves. Nonetheless the present paper represents an attempt to provide a framework through which such submodels may be combined to calculate energy partition. This may ultimately yield a more accurate method for calculating evaporation from sparse crops and, hence, a better understanding of how stomatal control takes over from soil conditions as crop cover increases.

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APPENDIX

Derivation of the sparse crop combination equation

Introducing Eq. (8) into Eqs. (9) and (10) gives

$$\lambda E_s = \frac{\Delta A_s + (\rho c_p / r_a^s) [D + \{\Delta A - (\Delta + \gamma) \lambda E\} r_a^a / \rho c_p]}{\Delta + \gamma(1 + r_s^s / r_a^s)} \quad (\text{A1})$$

$$\lambda E_c = \frac{\Delta(A - A_s) + (\rho c_p / r_a^c) [D + \{\Delta A - (\Delta + \gamma) \lambda E\} r_a^a / \rho c_p]}{\Delta + \gamma(1 + r_s^c / r_a^c)}. \quad (\text{A2})$$

The total evaporation flux, λE , is given by $\lambda E = \lambda E_s + \lambda E_c$. Adding Eqs. (A1) and (A2) gives

$$\lambda E = \frac{\Delta A_s r_a^s + \rho c_p [D + \{\Delta A - (\Delta + \gamma)\lambda E\} r_a^a / \rho c_p]}{(\Delta + \gamma)r_a^s + \gamma r_s^s} + \frac{\Delta(A - A_s)r_a^c + \rho c_p [D + \{\Delta A - (\Delta + \gamma)\lambda E\} r_a^a / \rho c_p]}{(\Delta + \gamma)r_a^c + \gamma r_s^c}. \quad (\text{A3})$$

Multiplying (A3) by the product of the two denominators and collecting terms in λE gives

$$\begin{aligned} \lambda E [& \{(\Delta + \gamma)r_a^s + \gamma r_s^s\} \{(\Delta + \gamma)r_a^c + \gamma r_s^c\} + (\Delta + \gamma)r_a^a \times \\ & \times \{(\Delta + \gamma)r_a^c + \gamma r_s^c\} + (\Delta + \gamma)r_a^a \{(\Delta + \gamma)r_a^s + \gamma r_s^s\}] \\ & = (\Delta A_s r_a^s + \rho c_p D + \Delta A r_a^a) \{(\Delta + \gamma)r_a^c + \gamma r_s^c\} + \\ & + \{\Delta(A - A_s)r_a^c + \rho c_p D + \Delta A r_a^a\} \{(\Delta + \gamma)r_a^s + \gamma r_s^s\}. \end{aligned} \quad (\text{A4})$$

If we define

$$R_s = (\Delta + \gamma)r_a^s + \gamma r_s^s \quad (\text{A5})$$

$$R_c = (\Delta + \gamma)r_a^c + \gamma r_s^c \quad (\text{A6})$$

$$R_a = (\Delta + \gamma)r_a^a \quad (\text{A7})$$

and substitute these into (A4) we get

$$\begin{aligned} \lambda E (R_s R_c + R_c R_a + R_s R_a) & = \{\Delta A (r_a^a + r_a^s) + \rho c_p D - r_a^s \Delta (A - A_s)\} R_c + \\ & + \{\Delta A (r_a^a + r_a^c) + \rho c_p D - r_a^c \Delta A_s\} R_s. \end{aligned} \quad (\text{A8})$$

Now

$$R_s + R_a = (\Delta + \gamma)(r_a^a + r_a^s) + \gamma r_s^s \quad (\text{A9})$$

and

$$R_c + R_a = (\Delta + \gamma)(r_a^a + r_a^c) + \gamma r_s^c \quad (\text{A10})$$

so we can write Eq. (A8) as

$$\lambda E (R_s R_c + R_c R_a + R_s R_a) = PM_s R_c (R_s + R_a) + PM_c R_s (R_c + R_a) \quad (\text{A11})$$

where

$$PM_s = [\Delta A + \{\rho c_p D - \Delta r_a^s (A - A_s)\} / (r_a^a + r_a^s)] [\Delta + \gamma \{1 + r_s^s / (r_a^a + r_a^s)\}]^{-1} \quad (\text{A12})$$

and

$$PM_c = [\Delta A + (\rho c_p D - \Delta r_a^c A_s) / (r_a^a + r_a^c)] [\Delta + \gamma \{1 + r_s^c / (r_a^a + r_a^c)\}]^{-1}. \quad (\text{A13})$$

So Eq. (A11) becomes

$$\lambda E = C_c PM_c + C_s PM_s \quad (\text{A14})$$

providing

$$C_s = R_c (R_s + R_a) / (R_s R_c + R_c R_a + R_s R_a) = \{1 + R_s R_a / R_c (R_s + R_a)\}^{-1} \quad (\text{A15})$$

$$C_c = R_s (R_c + R_a) / (R_s R_c + R_c R_a + R_s R_a) = \{1 + R_c R_a / R_s (R_c + R_a)\}^{-1}. \quad (\text{A16})$$

Equation (A14) is the desired sparse crop combination equation. The contributions λE_c and λE_s can now be computed from Eqs. (9) and (10) with D_0 given by Eq. (8).

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